Hydraulics I

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Course Content

• Properties of Fluids
• Hydrostatics
• Fluid Kinematics
• Basics of Fluid Dynamics
Chapter 1 Properties of Fluids

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Definitions

- **Fluids (liquids and gases)**: a substance which deforms continuously, or flows, when subjected to shear stress.
- **Fluid Mechanics**: The study of liquids and gasses at rest (statics) and in motion (dynamics)
- **Engineering applications**
  - Dams and reservoirs
  - Water supply pipelines
  - Groundwater movement
  - Runoff in parking lots
  - Pumps, filters, rivers, etc.
Fluid Properties

Density

Density: mass of substance per unit volume (kg/m³)

- Mass per unit volume (e.g., @ 20 °C, 1 atm)
  - Water \( \rho_{\text{water}} = 1000 \text{ kg/m}^3 \)
  - Mercury \( \rho_{\text{Hg}} = 13,500 \text{ kg/m}^3 \)
  - Air \( \rho_{\text{air}} = 1.22 \text{ kg/m}^3 \)

- Densities of liquids are nearly constant (incompressible) for constant temperature

- **Specific volume** = 1/density
Specific Weight

Specific Weight: the force exerted by the earth’s gravity up on a unit volume of substance

\[ \gamma = \rho g \quad [\text{N/m}^3] \]

- Weight per unit volume (e.g., @ 20 °C, 1 atm)

\[
\begin{align*}
\gamma_{\text{water}} &= (998 \text{ kg/m}^3)(9.807 \text{ m}^2/\text{s}) \\
&= 9790 \text{ N/m}^3 \\
\gamma_{\text{air}} &= (1.205 \text{ kg/m}^3)(9.807 \text{ m}^2/\text{s}) \\
&= 11.8 \text{ N/m}^3
\end{align*}
\]

Specific Gravity/Relative density

Ratio of fluid density to that of water at STP (@ 20 °C, 1 atm)

\[
\begin{align*}
SG_{\text{liquid}} &= \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{\rho_{\text{liquid}}}{9790 \text{ kg/m}^3} \\
SG_{\text{gas}} &= \frac{\rho_{\text{gas}}}{\rho_{\text{air}}} = \frac{\rho_{\text{gas}}}{1.205 \text{ kg/m}^3}
\end{align*}
\]

- Water \[ SG_{\text{water}} = 1 \]
- Mercury \[ SG_{Hg} = 13.6 \]
- Air \[ SG_{\text{air}} = 1 \]
Viscosity

• By the virtue of cohesion and interaction between fluid molecules offers resistance to relative motion (shear deformation).

• Newton’s law of viscosity: shear stress and viscosity

\[ \tau = \mu \frac{du}{dy} \]

\( \tau \) is shear stress N/m², \( \mu \) is coefficient of dynamic viscosity (Ns/m²), and \( \frac{du}{dy} \) velocity of gradient (radians/s)

Dynamic viscosity (\( \mu \)). It is the shear force per unit area required to drag one layer of fluid with unit velocity past another layer a unit distance away.

Unit = kg/m.s or N.s/m²

Poise (p) = 0.1 kg/m.s

Kinematic viscosity (\( \nu \)) defined as the ratio dynamic viscosity to mass density.

Unit: m²/s

Stokes (st) = 0.0001 m²/s
Newtonian and Non-Newtonian Fluids

Compressibility and elasticity

- Deformation per unit of pressure change

- Bulk modulus of elasticity $E_v = -\frac{dp}{dV/V} = \frac{dp}{dV/\rho}$

- For water $E_v = 2.2$ GPa,
  1 MPa pressure change = 0.05% volume change
  Water is relatively incompressible
**Vapor Pressure**

- Partial pressure of liquid escaping molecules
- Vapor pressure increases with temperature
- Pressure at which a liquid will boil for given temp.
- The saturated vapor pressure for water at 20°C is $2.45 \times 10^5$ N/m$^2$

![Diagram of vapor pressure](image)

**Surface Tension**

- Cohesion $\rightarrow$ small tensile forces at the interface between liquid and air called surface tension
- Adhesion $>$ Cohesion $\rightarrow$ capillary rise
- Adhesion $<$ cohesion $\rightarrow$ capillary depression
- $\sigma, \text{water}= 0.073$ N/m (@ 20°C)
Capillary Rise/depression

\[ \Delta h = \frac{4\sigma \cos \theta}{\gamma d} \]

Example 1

- **Given:** Pressure of 2 MPa is applied to a mass of water that initially filled 1000-cm\(^3\) volume. 
  E = 2.2x10\(^9\) Pa

- **Find:** Volume after the pressure is applied.

**Solution**

\[ E_v = -\frac{\Delta p}{\Delta V/V} \]
\[ \Delta V = -\frac{\Delta p}{E_v} V \]
\[ = -\frac{2 \times 10^6 \text{ Pa}}{2.2 \times 10^9 \text{ Pa}} \times 1000 \text{ cm}^3 \]
\[ = -0.909 \text{ cm}^3 \]
\[ V_{\text{final}} = V + \Delta V \]
\[ = 1000 - 0.909 \]
\[ V_{\text{final}} = 999.09 \text{ cm}^3 \]
Example 2

The density of an oil at 20°C is 850 kg/m³. Find its relative density and kinematic viscosity if the dynamic viscosity is 5 x 10⁻³ kg/ms.

Solution:

- Relative density, \( \sigma = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} \)
  
  \[
  = \frac{850}{10^3}
  \]
  
  \[
  = 0.85
  \]

- Kinematic viscosity, \( \nu = \frac{\mu}{\rho} \)

  
  \[
  = \frac{5 \times 10^{-3}}{850}
  \]
  
  \[
  = 5.88 \times 10^{-6} \text{m}^2/\text{s}
  \]

Example 3

If the velocity distribution of a viscous liquid (\( \mu = 0.9 \text{ Ns/m}^2 \)) over a fixed boundary is given by \( u = 0.68y - y^2 \) in which \( u \) is the velocity in m/s at a distance \( y \) metres above the boundary surface, determine the shear stress at the surface and at \( y = 0.34 \text{ m} \).

If the above velocity distribution occurred in a pipe of 3 cm diameter, find the total resistance over a length of 100 m.
Example 3 Solution

• $u = 0.68y - y^2$
• $\frac{du}{dy} = 0.68 - 2y$; hence $(\frac{du}{dy})_{y=0} = 0.68 \text{ s}^{-1}$
  and $(\frac{du}{dy})_{y=0.34} = 0$
• Dynamic viscosity of the fluid, $u = 0.9 \text{ Ns/m}^2$
• From Newton's equation
  $\tau = \mu(\frac{du}{dy})$, shear stress $(\tau)_{y=0} = 0.9 \times 0.68$
  $= 0.612 \text{ N/m}^2$
  and at $y = 0.34 \text{ m}$, $\tau = 0$.

Example 4

• **Find:** Capillary rise between two vertical glass plates 1 mm apart. $\sigma = 7.3 \times 10^{-2} \text{ N/m}$. L is into the page.
Assume $\cos \theta = 1$.

\[
\sum F_{vertical} = 0
\]
\[
2\sigma l - h \tan \gamma = 0
\]
\[
h = \frac{2\sigma}{\tan \gamma}
\]
\[
h = \frac{2 \times 7.3 \times 10^{-2}}{0.001 \times 9810}
\]
\[
h = 0.0149 m
\]
\[
h = 14.9 mm
\]
Example 5

• **Find:** The formula for the gage pressure within a spherical droplet of water?

• **Solution:** Surface tension force is resisted by the force due to pressure on the cut section of the drop.

\[ p(tau r^2) = 2\pi r \sigma \]

\[ p = \frac{2\sigma}{r} \]